Heterogenous Discount Rates and Optimal Portfolio Diversification

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Abstract

According to modern portfolio theory, no single investor can beat "the market". In other words, it is assumed that rational investors do not deviate from the market portfolio. The wide-spread use of market capitalisation-weighted indices as benchmarks for investors' portfolios shows that this assumption of modern portfolio theory currently prevails across the financial market. Although the prevailing view, this paper argues that the optimal investment strategy in fact differs across investors when discount rates are heterogeneous. Such different discount rates are the result of socio-economic factors that lead to heterogeneity in the valuation of risks, as well as heterogeneity in investors' time preferences. Using a stylised example, this paper shows that heterogeneous discount rates lead to a different portfolio allocation than "the market" would suggest. Furthermore, the results suggest that heterogenous discount rates have implications on long-term investments in the real-economy, such as investments in the low-carbon economy.

Key-Takeaways:

- The assumption that rational investors should not deviate from the market portfolio currently prevails across financial markets;
- However, given heterogenous discount rates through different time preferences and valuations of risk, the optimal investment strategy differs.
- Heterogenous discount rates also matter for capital allocation and affect the real economy

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1. INTRODUCTION

Modern portfolio theory, as pioneered by Markowitz (1952), Sharpe (1964), and Tobin (1958), suggests that an optimal investing strategy involves buying the market portfolio. An investor can optimise the mean-variance constraint using a market portfolio, assuming the presence of a risk-free asset. The market portfolio represents optimal diversification in these theories.

Based on the notion that an individual investor cannot beat the market, the overwhelming majority of investors use market-capitalisation indices as benchmarks for their portfolios. This paper argues that an individual investor might be better off not investing in the market portfolio, even in the presence of consistent beliefs about future cash flows. This analysis demonstrates that even based on the efficient market hypothesis, the market portfolio only represents the optimal portfolio in the particular case that the investor's discount rate is identical to that of the market, i.e. that discount functions are homogenous. However, the paper argues that if heterogenous discount rates are introduced to the model, the market portfolio is not optimal for investors with a different discount rate, even under the assumption that markets are otherwise informationally efficient and investors are rationally time-consistent, as defined by Fama (1965, 1970). The heterogeneity of discount rates creates systematic sub-optimal returns for investors when following "the market".

This paper contributes to the existing literature by showing that it is rational for individual investors to deviate from the market portfolio if they have a different discount function than the market. For example, if long-term paybacks are more uncertain due to trust issues or external uncertainties, it may be rational (and utility-maximising) to prefer a short-term payoff. Similarly, long-term liabilities will create a higher valuation of future cash flows. This paper combines empirical evidence on the existence of heterogenous discount rates with stylised examples and practical implications of heterogenous discount rates to make the case that an alternative choice to "the market" would be rational.

The paper first gives a summary of the current use of modern portfolio theory in the financial markets (Section 2). It seeks to collect evidence for the presence of heterogenous discount functions between financial market actors due to the heterogeneity in value of risks and time preferences (Section 3). It then shows using a simple simulation that returns for such investors discounting the future differently than the market are sub-optimal if they buy the market portfolio (Section 4). The paper will bridge the theoretical assessments to the practice and look at implications for long-term institutional investors (Section 5). The paper concludes that considering alternative notions of valuing the future is necessary to achieve optimal diversification for investors with heterogenous discount rates (Section 6).

2. CURRENT USE OF MODERN PORTFOLIO THEORY

Before looking into the notion of heterogenous discount rates, modern portfolio theory and its current use will be briefly summarised. While Markowitz may be the father of modern portfolio theory, it is the work of Tobin (1958) and Sharpe (1964) that turn this conceptual framework into a dominant force in financial markets. Whereas Markowitz argues that there are a range of efficient portfolios on the 'efficient frontier', whose adoption is driven by the risk preferences of the investor, the 'Tobin separation theorem' proves that, assuming for the existence of a risk-free asset (such as cash or Treasury bonds), there is only one 'super-efficient' portfolio. Investors then, based on their risk preferences, simply adjust the ratio between the 'super-efficient' portfolio and the risk-free asset. Sharpe (1964) then proves mathematically that this super-efficient portfolio is the market portfolio – a portfolio holding all the world's assets. This is proven using a model called the "capital asset pricing model (CAPM)", which reduces the challenge set by Markowitz (that of defining the correlation of all assets with each other) to a simpler version, where not the correlation with each other, but with the market risk (beta) is what needs to be measured. Diversification then ensures the elimination of idiosyncratic risk.

This model is challenged in the literature, notably by adding other factors, for example, low price to earnings ratio (Basu 1977), low book-to-market ratios (Chan, Jegadeesh, and Lakonishok 1996; Barber and Lyon 1997), leverage (Bhandari 1998), and short-term price momentum (Jegadeesh 1990). Partly as a function of this criticism, the CAPM has seen further developments, notably in the form of the intertemporal capital asset pricing model (Merton 1973) and arbitrage pricing theory (Ross 1976). Fama and French (1993) develop a three-factor and a five-factor model (1996) when integrating bonds, which, the authors claim, ultimately unifies these model advances based on the simple idea that CAPM works, albeit with more than just a market factor (2004).[1]

Indeed, while intellectually and academically the key tenets of modern portfolio theory are challenged, the underlying notion, that the market portfolio represents optimal diversification, remains prominent. Market-capitalisation weighted indices, seen as representing the market, dominate financial markets. These indices, based on the market capitalisation (share price times number of outstanding stocks), are assumed to represent the market portfolio as market prices, or in this case, the associated values of companies, are seen to reveal market information and thus the collective market opinion on companies. Market-capitalisation-weighted indices are one of the most prominent benchmark tool for investors (S&P Global, 2020). Research suggests that e.g. most funds use benchmark indices as sector allocation guidelines (Thomä et al. 2014).

3. HETEROGENOUS DISCOUNT RATES

Given the focus of this study on discount rates, it is worth briefly highlighting the core principles behind discount functions. A discount function describes the relative weighting of rewards received at different points in time (Frederick, Loewenstein, and O'Donoghue 2002). One of the main tenets of the rational choice theory in terms of utility-maximisation is the time-

[1] Naturally, there remain a range of fundamental criticisms, notably based on the idea of market inefficiency (Rosenberg 1976), from the champions of chaos theory arguing against the notion of normal distribution of returns (Mandelbrot 2004) and behavioral economics. Falkenstein (2012) for example argues that a missing 'risk premium' because of human irrationality makes low-risk stocks a better investment.

consistency of preferences by economic agents. This tenet postulates that individuals have consistent preferences over time, in other words, a "no regret" position at point $t+1$ relative to their choices at point t, expressed as

$$f(D) = \frac{1}{(1+r)^n}, \tag{1}$$

where r is the interest rate (which discounts the cash flow or any other utility or good received by the economic actor) and n the number of periods.

If this was not the case, the utility would not be maximised inter-temporally. Mathematically, this implies that individuals have an exponential discount function

$$f(D) = e^{-r}. \tag{2}$$

Most economic modelling and analysis are based on the premise that "we do not suppose time to be allowed for any alteration in the character or tastes of the man himself." (Marshall 1920, 79). In other words: "What is assumed is that consumers are fairly consistent in their tastes and actions – that they do not flail around in unpredictable ways, making themselves miserable by persistent errors of judgment or arithmetic." (Nordhaus and Samuelson 1995, 78). A prominent example of the application of an exponential discount function is by Becker and Murphy (1988) who attempt to show the rationality and time-consistency of addiction.

Note that this assumption is highly criticised. As crucial as this condition for the theory of utility-maximising agents, as weak is its theoretical and empirical foundation. In practice, economic agents discount the future in a non-exponential way. Instead, other mathematical functions are more relevant, the most prominent being the hyperbolic discount function (Thaler 1981; Laibson 1997; Frederick, Loewenstein, and O'Donoghue 2002). The hyperbolic discount function suggests economic agents have 'present-biased preferences', where the immediate future is discounted highly, but the long-term future progressively at a lower rate. In terms of finance, this is important because it suggests that short-term payoff optimization is valued higher than long-term payoffs. Thus, investments with higher discounted cash flows, but longer payback periods (or more consistent but lower payback periods over the life of a project), may be rejected for projects with higher returns in the short-term, but lower returns in the long-run, even if the aggregate non-discounted returns are higher in the first case.

Crucially, the presence of hyperbolic discount functions of an individual economic agent suggests sub-optimal economic decision-making. This literature is well established and there are arguments on either side. This paper posits however that even in the presence of time-consistent preferences in finance, investment allocation decisions may sub-optimal if there is heterogeneity between economic agents with regard to their time preferences. In the following, this paper states two reasons for heterogenous discount rates among investors. There is first heterogeneity in the "valuation" of risk (Section 0) – as expressed in investment beta or the delta between the discount and risk-free rate – and second investors have (rationally) heterogeneous time preferences (Section 3.2). Thus, both factors – the heterogeneity in the valuation of risk and the heterogeneity of time preferences imply that in practice investors have heterogenous discount rates and it would be irrational for them to follow the homogenous market discount rate.

3.1. Heterogeneity in the Valuation of Risk

Before reviewing the literature, it is worth briefly highlighting this paper assumes an endogenous formation of interest rates, based on individual behaviour functions. Readers may believe to have identified a fundamental misunderstanding of the role of discount rates in price formation and financial modelling, where such rates represent an expression of the riskiness of an asset and the risk-free rate (exogenous interest rates), rather than an expression of time preferences, themselves a product of idiosyncratic utility functions (endogenous interest rates). As outlined below, such a narrow definition does not reflect the current understanding of the literature and will thus not be further explored.

In finance, risk preferences reflect investors' tolerance for systematic and concentrated market asymmetries with the revealed premium of these risks assumed to indicate the associated discount rate and the investments beta. Researchers have extensively documented investors' revealed risk tolerance, which can be arbitrarily influenced by exogenous factors such as weather or sporting events (Saunders 1993; Hirshleifer and Shumway 2003; Edmans, Garcia, and Norli 2007).

As Andreoni and Sprenger (2012) note, "time and risk are intertwined" — an individual's risk aversion towards the future uncertainties triggers a bias for the present. Furthermore, Jacobs-Lawson and Douglas (2005), Andreoni and Sprenger (2012), and Tanaka, Camerer, and Nguyen (2010) find that risk tolerance influences time preferences. Risk, therefore, mediates individuals' time preferences, and in the context of this paper, acts as a proxy to at least explore, the possibility of heterogeneous discount functions among investors.

Given that risk tolerance differs, it is logical to assume investors must discount the future differently. Reasons for the different risk preferences of investors are a variety of socio-economic characteristics shaping those preferences. Risk-taking behavior is heavily affected by socio-economic factors as e.g. country of birth, time, gender, cognitive abilities, genetic factors and major shocks (Ayton et al. 2020).

For example, through the allocation of sampled investors' assets, Wang and Hanna (1997) conclude that risk tolerance increases with age, while Hallahan, Faff, and McKenziea (2004) and Morin and Fernandez (1983) come to the opposite verdict. A new research stream examines life experiences and their effect on risk behaviour. Ayton et al. (2020) present the results of six different papers all emphasising the effects of experience as, for example, recessions, wars, but also natural disasters. Abayato and Lynham (2020) look at fishers in Asia and investigate the effect of a Typhoon on individual risk preferences. By weakening food security through demolishing reels and fish population the typhoon had a large impact on specific communities in which fishers disclosed a changed risk behaviour afterwards – they are more risk-averse. Olbrich, Quaas, and Baumgärtner (2011) examine the effects of heterogeneous precipitation risks on the risk behaviour of Namibian farmers and identify a correlation contingent on previous experiences with precipitation risks. Cohn et al. (1975) observe that wealthy investors partake in riskier investments in a cross-section of American households. Dulebohn (2002) and Hallahan, Faff, and McKenziea (2004) note that gender, retirement plan participation, self-efficacy, knowledge of investment principles, and general risk propensity all play a role. Furthermore, Ayton et al. (2020) present the results of six different papers all emphasising the effects of experience as e.g. recessions, wars, but also natural disasters.

To conclude, the most important inference from this section is that risk preferences of investors can be shaped by socio-economic factors that influence different economic agents differently and thus lead to a heterogenous valuation of risk. Since risk can be used as a good proxy for individuals' time preferences, such heterogeneity in the valuation of risks leads to heterogenous time preferences, leading to heterogenous discount rates.

3.2. Heterogeneity in Time Preferences

Even if the proxy of risks for time preferences fails – in other words where representations of risk as they materialise in discount rates are homogenous - time preferences in the spirit of Irving Fisher's Theory of Interest (1930) will still lead to heterogenous outcomes.

While scholarly consideration of intertemporal choice extends back to the work of Adam Smith, John Raes, and Eugen von Böhm-Bawerk (Palacios-Huerta 2003), looking at other disciplines, the contemporary psychologist such as Thaler (1981) who seeks to empirically quantify individual discount rates (Loewenstein and Elster 1992). The intertemporal discount literature presents a complex and nuanced decision framework. Warner and Pleeter (2001), and Frederick, Loewenstein, and O'Donoghue (2002) find that individual discount rates change dramatically across studies from 0 to 70 % per year. Furthermore, as illustrated by the emergence and proliferation of Behavioral Finance and Economic research, the role of human psychology in economic transactions has important implications for both policymakers and practitioners (Barberis and Thaler 2003; Shiller 2003). For example, several studies note the effect of wealth with higher incomes associated with longer time horizons (Harrison, Lau, and Williams 2002; Warner & Pleeter 2001). Furthermore, heterogenous time preferences are an important assumption in a lot of disciplines, for example also in the climate literature (Layton and Brown 2000).

Coming to finance, many if not the majority of institutional investors may in practice be short-term investors as a result of short-term profit maximisation (Hazen 1991), whereas some institutional investors such as pension funds and life insurance companies or endowments, however, might have longer time preferences due to long-term liabilities (Della Croce, Stewart, and Yermo 2011).

However, despite the ample evidence that different liability structures and different incentives of investors but also the literature from other disciplines imply that different time-horizons matters in the discounting of the future, no research to date has sought to explore the reason and formation of such heterogeneity further. So, the question is how heterogenous time preferences are measured that, in turn, result in a heterogenous discount rate. Yet since the emergence of the Efficient Market Hypothesis, Hirshleifer (2001) and Dempsey (2013) note the failure of academic finance to consider investor psychology. Asset pricing models generalise investors to be a homogenous group of rational, utility-maximising, linear-time-discounting agents (Dempsey 2013; Berk 1997). Nevertheless, this has important implications as Thomä and Chenet (2017) observe — "investors may not optimise inter-temporal returns" with standard financial models and therefore inefficiently price future risks. In both stocks and bonds, Campbell and Shiller (1987) find significant deviations from the theoretical spread implied by the present value calculation of financial models.

4. OPTIMAL DIVERSIFICATION WITH HETEROGENOUS DISCOUNT RATES

The question now is what implications heterogenous discount rates have with regard to optimal diversification. Such implications can be illustrated using a simple stylised portfolio consisting of seven companies (the size of the portfolio does not influence the conclusions, the exercise can be replicated with any number of companies). The paper assumes that asset prices are determined using the discounted cash flow model

$$NPV_i = \sum_{n=1}^{t} \frac{CF_n}{(1+r)^n}, \tag{3}$$

where

In this stylised portfolio, all seven companies (A-G) have cash flows at t = 0 of 100. Cash flows for these seven companies are modelled out to the year 2050. One company has zero cash flow growth (company A), three companies see their cash flows increase (alternatively by 3 – company B, 6 – company C, and 9%– company D) and three companies see their cash flows decrease by 3 (company E), 6 (company F), and 9% (company G). For this stylised example, let us assume that the "market" interest rate r is 10% and the interest rate of the heterogenous investor is 5%. Following Equation $fD = \frac{1}{(1+r)^n}$,

(1)

where *r* is the interest rate (which discounts the cash flow or any other utility or good received by the economic actor) and *n* the number of periods.

If this was not the case, the utility would not be maximised inter-temporally. Mathematically, this implies that individuals have an exponential discount function

the net present value of companies A-G can be found in **Error! Reference source not found.**

Exhibit 1: Net present value of companies A-G modelled out to 2050			
Company	**Net present value (in €) with 10% discount rate**	**Net present value (in €) with 5% discount rate**	**Actual cash flows without discounting**
A	1042.69	1637.25	3100.00
B	1366.75	2357.69	5000.27
C	1877.75	3586.49	8480.17
D	2712.17	5740.32	14957.52
E	829.01	1200.00	2036.74
F	682.24	923.65	1421.87
G	577.33	741.12	1051.40
Sum	9087.93	16186.51	36047.97

Assuming the seven companies collectively represent all market assets and thus following

$$NPV_p = \sum NPV_i \, , \qquad (4)$$

where *NPV(p)* is the summed net present value of the whole portfolio (all companies A-G) and *NPV(i)* is the net present value of each company A-G, the market portfolio for the investor with a discount rate of 5% values the overall portfolio €16,186.51, whereas the market with a discount rate of 10% only values it €9,087.93.

Note that the actual cash flows are in both cases the same over time. In question is not a heterogeneity of assumptions about future cash flows or even the accuracy of these assumptions, but rather how those future cash flows are valued today.

With

$$W_i = NPV_i / NPV_p, \qquad (5)$$

where *W* is the weight of company *i* in the portfolio, *NPV(i)* is the net present value of the company and *NPV(p)* the net present value of the whole portfolio (all companies A-G), the optimal allocation to each company (A-G) can be calculated.

Exhibit 2: Optimal allocation to each company (A-G)

Company	Optimal diversification 10% discount rate	Optimal diversification 5% discount rate
A	11%	10%
B	15%	15%
C	21%	22%
D	30%	36%
E	9%	7%
F	8%	6%
G	6%	5%

The implication of the different valuation becomes clear when looking at **Error! Reference source not found.** The table shows the optimal allocation to each company (labelled A-G here) given a 5% and 10% discount rate. For example, using traditional modern portfolio theory, the optimal allocation to Company A for an investor with a discount rate of 10% would be 11% of their portfolio. For an investor with a discount rate of 5%, the optimal exposure would be 10%.

This stylised example demonstrated that heterogenous discount rates imply that there might be a better diversification of portfolio than following the market diversification. Note that it may be argued that the concept of net present value is fictitious. It is not the valuation of the future, but the realised returns that matter. However, in this case, long-term investors with lower discount rates face even higher sub-optimal returns, as companies with lower long-term returns are penalised less by valuations associated with higher discount rates, given that such rates reduce the penalty for companies with poor long-term performance.

While the percentage differences are not dramatic, the overall value effect is, as shown by the significant delta in the net present value of the market portfolio across the two investors.

5. CONSEQUENCES FOR CAPITAL ALLOCATION IN THE REAL ECONOMY

The conclusion so far is that there are many investors with potentially a range of different discount rates. The market discount rate is therefore not a homogenous discount function across all investors, but rather the average of all discount rates weighted by the size of the investor in the marketplace. Thus, by extension, any individual investor subsumed in the market investor category could find that his or her discount function differs from the market function. The results may in the first instance be relevant for investors seeking to optimise the mean-variance of their investments. However, in the second instance, such an output also has more fundamental implications for capital allocation in the real economy.

If we assume that the market discount rates are higher than the discount rates for those investors valuing the long-term more than the market, taking a homogenous discount rate for granted might become problematic for long-term investment. For example, renewable energy has a lower share of operating costs, but a higher share of capital costs compared to fossil fuel energy. Consequently, the payoffs are likely to be more long-term with longer payback periods (Thomä and Dupré 2014).

Building on the outlined example above, consider six companies A-LC, B-LC, C-LC and E-CI, F-CI, G-CI, where A is the letter of the company and LC stands for low carbon and CI for carbon-intensive indicating if the company belongs to a low carbon sector or a high carbon-intensive sector. Furthermore, assume that in the first ten years companies from the low carbon sector receive steadily €100, however after ten years the renewable energy is getting profitable, i.e. cash flows are increasing (for company A-LC 3, company B-LC 6, and company C-LC 9%). In contrast, the carbon-intensive companies realised high returns in the first 10 years (for company E-CI 3, company F-CI 6, and company G-CI 9%) but then through the implementation of carbon tax stagnate the cashflows with 100.

Exhibit 3: Implications of heterogenous discount rates for the capital allocation of the real economy

Low in carbon			Carbon intensive		
C	**Optimal diversification 10% discount rate**	**Optimal diversification 5% discount rate**	**C**	**Optimal diversification 10% discount rate**	**Optimal diversification 5% discount rate**
A-LC	15.1%	15%	**D-CI**	14.8%	13.7%
B-LC	16.9%	18.1%	**E-CI**	16.1%	14.7%
C-LC	19.6%	22.8%	**F-CI**	17.5%	15.8%
Sum	51.6%	55.9%		48.4%	44.1%

Note: C = Company
Source: Own calculation and representation

The results can be found in **Error! Reference source not found.** Regardless of the discount rate, there would be more investment in the low-carbon sectors than in the carbon-intensive sectors. However, the example here shows that having a discount rate from 5%, which means that the long-term is valued higher than with a discount rate of 10%, we will end up having about 4% more investment in the low-carbon economy. This rather simple stylised example demonstrates that the different discount rates matter also in terms of capital allocation in the real economy. With a small discount rate that values the future higher than the market a better capital allocation for the policy aim of having a transition to a low-carbon economy.

6. CONCLUSION

This paper argues that the optimal diversification of a portfolio differs between investors if we assume that they value the long term differently from the short term, i.e. if they have heterogeneous discount rates. For an individual investor who has a different discount rate than the “market”, it might be suboptimal to follow the market allocation of capital. In contrast to other criticisms of the efficient market hypothesis, which emphasise that the assumptions may be unrealistic because investors may be irrational, this paper demonstrates that an individual investor's decision is still rational. This reasoning also holds under the time-consistent premise. This paper also posits that heterogeneous discount rates also matter for capital allocation and affect the real economy.

If investors value the long term more than the market, they might also invest more in low-carbon sectors, as returns will increase over time rather than in the immediate future. Therefore, the paper concludes that it is not only important to focus on heterogeneous discount rates when investors want to optimise their investments, but also when long-term investments are required, such as investment in a low-carbon economy. In this paper, however, one particular issue is not addressed: the identifiability of heterogeneous discount rates. As Section 3 showed the formation of a discount rate is a very individual aspect based on socio-economic factors or time preferences. Nevertheless, it is not yet clear how investors can decide on their individual discount rate and know how it deviates from the market discount rate. This paper highlights that it is worth investing to understand more about the individual formation of discount rates and further research needs to be done here.

7. REFERENCES

Andreoni, James and Charles Sprenger. 2012. “Risk preferences are not time preferences”. In *American Economic Review*. Vol. 102. No. 7. 3357-3376. https://www.jstor.org/stable/41724637.

Ayton, Peter, Gennaro, Bernile, Bucciol, Alessandro and Luca Zarri. 2020. “The impact of life experiences on risk-taking”. In *Journal of Economic Psychology*. Vol. 79. No. 102274. https://doi.org/10.1016/j.joep.2020.102274.

Bachelier, Loius. 1900. “Théorie de la speculation”. In *Annales scientifiques de l’Ecole Normale Supérieure*. Vol. 3. No. 17. 21-86.

Barber, Brad. M. and John D. Lyon. 1997. “Detecting long-run abnormal stock returns: The empirical power and specification of test statistics”, In *Journal of Financial Economics*, Vol. 43. No. 3. 341-372. https://doi.org/10.1016/S0304-405X(96)00890-2.

Barberis, Nicholas and Richard Thaler. 2003. “A survey of behavioral finance”. In *Handbook of the Economics of Finance*. Vol. 1B, 1053-1128. https://doi.org/10.1016/S1574-0102(03)01027-6.

Basu, Sanjoy. 1977. “Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A test of the Efficient Market Hypothesis*”*. In *The Journal of Finance*. Vol. 32. No. 3. 663-682. https://doi.org/10.2307/2326304.

Becker, Gary S. and Kevin M. Murphy. 1988. A Theory of Rational Addiction, In *Journal of Political Economy*. Vol. 96. No. 4, 675-700. https://www.jstor.org/stable/1830469.

Berk, Jonathan B. 1997. “Necessary conditions for the CAPM”. In *Journal of Economic Theory*. Vol. 73. No. 1. 245-257. https://doi.org/10.1006/jeth.1996.2218.

Bhandari, Laxmi C. 1988. “Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence”. In *The Journal of Finance*. Vol. 43. No. 2. 507-528. https://doi.org/10.2307/2328473.

Campbell, John. Y. and Robert J. Shiller. 1987. “Cointegration and tests of present value models”. In *Journal of Political Economy*. Vol. 95. No. 5. 1062-1088. https://www.jstor.org/stable/1833129.

Chan, Louis K. C., Jegadeesh, Narashiman and Josef Lakonishok. 1996. “Momentum Strategies”. In *The Journal of Finance*. Vol 7. No. 5. 1681-1713. https://doi.org/10.1111/j.1540-6261.1996.tb05222.x.

Cohn, Richard A., Lewellen, Wilbur G., Lease, Ronald C. and Gary G. Schlarbaum.1975. “Individual Investor Risk Aversion and Investment Portfolio Composition”. In *The Journal of Finance*. Vol. 30. No. 2. 605-620. https://doi.org/10.1111/j.1540-6261.1975.tb01834.x.

Della Croce, Raffaele, Stewart, Fiona and Juan Yermo. 2011. “Promoting longer-term investment by institutional investors: selected issues and policies”. In *OECD Journal: Financial Market Trends 2011* No. 1. 145-164. https://doi.org/10.1787/fmt-2011-5kg55b0z1ktb.

Dempsey, Mike. 2013. “The capital asset pricing model (CAPM): the history of a failed revolutionary idea in finance?”. In *Accounting Foundation*. Vol. 49. 7-23. https://doi.org/10.1111/j.1467-6281.2012.00379.x.

Dulebohn, James H. 2002. “An investigation of the determinants of investment risk behavior in employer-sponsored retirement plans”. In *Journal of Management*. Vol. 28. No. 1. 3-26. https://doi.org/10.1016/S0149-2063(01)00132-5.

Edmans, Alex, Garcia, Diego and Øyvind Norli. 2007. “Sports Sentiment and Stock Returns”. In *The Journal of Finance*. Vol. 62. No. 4. 1967-1998. https://doi.org/10.1111/j.1540-6261.2007.01262.x.

Falkenstein, Eric G. 2012. *The Missing Risk Premium: Why Low Volatility Investing Works*, CreateSpace Independent Publishing Platform, California, USA.

Fama, Eugene F. 1965. “The Behavior of Stock-Market Prices”. In *The Journal of Business*. Vol. 38. No. 1. 34-105. https://www.jstor.org/stable/2350752.

Fama, Eugene F. 1970. “Efficient Capital Markets: A Review of Theory and Empirical Work”. In *The Journal of Finance*. Vol. 25. No. 2. 383-417. https://doi.org/10.2307/2325486.

Fama, Eugene F. and Kenneth R. French. 1972. “Common risk factors in the returns on stocks and bonds”. In *Journal of Financial Economics*. Vol. 33. No. 1. 3-56. https://doi.org/10.1016/0304-405X(93)90023-5.

Fisher, Irving. 1930. *The Theory of Interest*. The Macmillan Company. New York. USA.

Frederick, Shane, Loewenstein, George and Ted O'Donoghue. 2002. “Time Discounting and Time Preference: A Critical Review”. In *Journal of Economic Literature*. Vol. 40. No. 2. 351-401. https://www.jstor.org/stable/2698382.

Hallahan, Terrence. A., Faff, Robert W. and Michael M. McKenzie. 2004. “An empirical investigation of personal financial risk tolerance”. In *Financial Services Review*. Vol. 13. No. 1. 57-78.

Harrison, Glenn W., Lau, Morten I. and Melonie B. Williams. 2002. “Estimating individual discount rates in Denmark: A field experiment”. In *American Economic Review*. Vol. 92. No. 5. 1606-1617.

Hazen, Thomas L. 1991. “Short-Term/Long-Term Dichotomy and Investment Theory: Implications for Securities Market Regulation and for Corporate Law”. In N.C.L. Rev., Vol. 70, no. 137. https://scholarship.law.unc.edu/nclr/vol70/iss1/11.

Hirshleifer, David and Tyler Shumway. 2003. “Good day sunshine: Stock returns and the weather”. In *The Journal of Finance*. Vol. 58. No. 3. 1009-1032. https://doi.org/10.1111/1540-6261.00556.

Hirshleifer, David. 2001. “Investor psychology and asset pricing”. In *The Journal of Finance*. Vol. 56. No. 4. 1533-1597. https://doi.org/10.1111/0022-1082.00379.

Jacobs-Lawson, Joy M. and Douglas A. Hershey. 2005. “Influence of Future Time Perceptive, Financial Knowledge, and Financial Risk Tolerance on Retirement Saving Behaviors”, In *Financial Services Review*. Vol 14. no. 4.

Jegadeesh, Narashiman. 1990. “Evidence of Predictable Behavior of Security Returns”. In *The Journal of Finance*. Vol. 45. No. 3. 881-898. https://doi.org/10.2307/2328797.

Laibson, David. 1997. “Golden eggs and hyperbolic discounting”. In *The Quarterly Journal of Economics*. Vol. 112. No. 2. 443-478. https://www.jstor.org/stable/2951242.

Layton, David F. and Gardner Brown. 2000. “Heterogeneous Preferences Regarding Global Climate Change”. In *Review of Economics and Statistics*. Vol. 82. No. 4. 616-624. https://www.jstor.org/stable/2646656.

Loewenstein, George and Jon Elster. 1992. *Choice over time*. Russell Sage Foundation. New York. USA.

Mandelbrot, Benoit B. 2004. “Fractals and Chaos: The Mandelbrot Set and Beyond”. Springer. New York. USA.

Markowitz, Harry. 1952. “Portfolio Selection”. In *The Journal of Finance*. Vol. 7. No. 2. 77-91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x.

Marshall, Alfred. 1920. *Principles of Economics*. 8th Edition. Macmillan and Co. Ltd. London. UK.

Merton, Robert C. 1973. “An Intertemporal Capital Asset Pricing Model”. In *Econometrica*. Vol. 41. No. 5. 867-887. https://doi.org/10.2307/1913811.

Morin, Roger A. and A. Fernandez Suarez. 1983. “Risk aversion revisited”. In *The Journal of Finance*. Vol. 38. No. 4. 1201-1216. https://doi.org/10.2307/2328020.

Nordhaus, William and Paul Samuelson. 1995. *Economics*. 15th Edition. McGraw-Hill Publishing Company. New York. USA.

Olbrich, Roland, Quaas, Martin F., and Stefan Baumgärnter. 2018. “Characterizing commercial cattle farms in Namibia: risk, management and sustainability“. *Working Paper Series in Economics*. No. 248. University of Lüneburg.

Osborne, Matthew F. M. 1959. “Brownian Motion in the Stock Market”. In *Operations Research*. Vol. 7. No. 2. 145-273. https://www.jstor.org/stable/167153.

Palacios-Huerta, Ignacio. 2003. “Time-Inconsistent Preferences in Adam Smith and David Hume”. In *History of Political Economy*. Vol. 35. No. 2. 241-268. https://doi.org/10.1215/00182702-35-2-241.

Rosenberg, Nathan. 1976. *Perspectives on Technology*. Cambridge University Press. Cambridge, UK.

Ross, Stephen A. 1976. “The arbitrage theory of capital asset pricing”. In *Journal of Economic Theory*. Vol. 13. No. 3. 341-360. https://doi.org/10.1016/0022-0531(76)90046-6.

S&P Global. 2020. "Why Cap Weighting?". Available at: https://www.spglobal.com/en/research-insights/articles/why-cap-weighting.

Samuelson, Paul A. 1965. "Proof that Properly Anticipated Prices Fluctuate Randomly". In *Industrial Management Review*. Vol. 6. No. 2. 41-49.

Saunders, Edward M. 1993. "Stock prices and Wall Street weather". In *The American Economic Review*. Vol. 83. No. 5. 1337-1345. https://www.jstor.org/stable/2117565.

Sharpe, William F. 1964. "Capital Asset Prices. A Theory of Market Equilibrium under Conditions of Risk". In *The Journal of Finance*. Vol. 19. No. 3. 425-442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x.

Shiller, R. J. (2003) From efficient markets theory to behavioral finance, Journal of Economic Perspectives, Vol. 17, No. 1, pp. 83-104. https://www.jstor.org/stable/3216841.

Tanaka, Tomomi, Camerer, Colin F. and Quang Nguyen. 2010. "Risk and time preferences: Linking experimental and household survey data from Vietnam". In *American Economic Review*. Vol. 100. No. 1. 557-571. https://www.jstor.org/stable/27804941.

Thaler, Richard. 1981. "Some empirical evidence on dynamic inconsistency". *Economics Letters*. Vol. 8. No. 3. 201-207. https://doi.org/10.1016/0165-1765(81)90067-7.

Thomä, Jakob and Stan Dupre. 2014. "The Turtle becomes the Hare – The implications of artificial short-termism for climate finance". *Discussion Paper 2° Investing Initiative*. https://2degrees-investing.org/wp-content/uploads/2014/10/The-turtle-becomes-the-hare-the-implications-of-artificial-short-termism-for-climate-finance-October-2014-1.pdf.

Thomä, Jakob, and Hugues Chenet. 2017. "Transition risks and market failure: A theoretical discourse on why financial models and economic agents may misprice risk related to the transition to a low-carbon economy". In *Journal of Sustainable Finance & Investment*. Vol. 7. No. 1. 82-98. https://doi.org/10.1080/20430795.2016.1204847.

Thomä, Jakob, Dupré, Stan, Gorius, Alexandre, Coeslier, Manuel and Danyelle Guyatt. 2014. "Optimal Diversification and the Energy Transition". *Research Report 2° Investing Initiative*. https://2degrees-investing.org/wp-content/uploads/2017/04/Optimal-diversification-and-the-energy-transition-impact-of-equity-benchmarks-on-portfolio-diversification-and-climate-finance-2014.pdf.

Tobin, James. 1958. "Liquidity Preference as Behavior Towards Risk". In *Review of Economic Studies*. Vol. 25. No. 2. 65-86. https://doi.org/10.2307/2296205.

Wang, Hui and Sherman D. Hanna. 1997. "Does risk tolerance decrease with age?". In *Financial Counselling and Planning*. Vol. 8. No. 2. http://dx.doi.org/10.2139/ssrn.95489.

Wang, Mei, Rieger, Marc O. and Thorsten Hens. 2016. "How time preferences differ: Evidence from 53 countries". In *Journal of Economic Psychology*. Vol. 52. 115-135. https://doi.org/10.1016/j.joep.2015.12.001.

Warner, John T. and Saul Pleeter. 2001. "The personal discount rate: Evidence from military downsizing programs". In *American Economic Review*. Vol. 91. No. 1. 33-53. https://www.jstor.org/stable/2677897.